2.6 - A Numerical Method (Euler's Method)

Recall the equation of the tangent line of the solution curve for the differential equation $y' = f(x, y)$, $y(x_0) = y_0$ is $y = y_0 + f(x_0, y_0)(x - x_0)$.

Definition: This is known as the **linearization of f at (x_0, y_0)**.

This process continues, and we can generalize the results:

$x_{n+1} = x_n + h$

$y_{n+1} = y_n + hf(x_n, y_n),\ n = 0, 1, 2, \ldots$

Example: Use Euler's method to obtain a four-decimal approximation of the indicated value. Carry out the recursion by hand, first using $h = 0.1$ and then using $h = 0.05$.

$y' = x + y^2, \qquad y(0) = 0; \qquad y(0.2)$

Example: Use Euler's method to obtain a four-decimal approximation of the indicated value. First use $h = 0.1$ and then use $h = 0.05$. Find an explicit solution for each initial-value problem and then construct a table displaying estimated value, actual value, absolute error, and percentage relative error.

$y' = 2xy, \qquad y(1) = 1; \qquad y(1.5)$